4 – SEQUENCES AND SERIES

EXERCISE 4.1

1. Verify whether the following sequences are G.P. If so, find tn.

i) 2,6,18,54,…….

Ans. $t_1 = 2, t_2 = 6, t_3 = 18, t_4 = 54, \ldots.$

Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = 3$

Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.

Here. a = 2, r = 3

$t_n = ar^{n-1}$

$\therefore t_n = 2(3^{n-1})$

ii) 1, –5, 25, –125 ……..

Ans. $t_1 = 1, t_2 = -5, t_3 = 25, t_4 = -125,$

Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = 5$

Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.

Here, a = 1, r = –5

$t_n = ar^{n-1}$

$\therefore t_n = (-5)^{n-1}$

iii) $\sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, \ldots\ldots$

Ans. $t_1 = \sqrt{5}, \ t_2 = \frac{1}{\sqrt{5}}, t_3 = \frac{1}{5\sqrt{5}} \ t_4 \frac{1}{25\sqrt{5}}, \ldots..$

Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \frac{1}{5}$

Since, the ratio of any two consecutive terms is a constant terms is a geometric progression.

Here, $a = \sqrt{5}, r = \frac{1}{5}$

$t_n = ar^{n-1}$

$\therefore\ t_n = \sqrt{5}\ \left(\frac{1}{5}\right)^{n-1}$

$= \quad (5)^{\frac{1}{2}}\ \ (5)^{1-n}$

$= \quad (5)^{\frac{3}{2}-n}$

iv) 3, 4, 5, 6,……

Ans. $t_1 = 3,\ t_2 = 4,\ t_3 = 5, t_4 = 6$ …

Here, $\frac{t_2}{t_1} = \frac{4}{3}, \frac{t_3}{t_2} = \frac{5}{4}, \frac{t_4}{t_3} = \frac{6}{5}$

Since, $\frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$

∴ The given sequence is not a geometric progression.

v) 7, 14, 21, 28,……

Ans. $t_1 = 7,\ t_2 = 14,\ t_3 = 21, t_4 = 28$ …

Here, $\frac{t_2}{t_1} = 2, \frac{t_3}{t_2} = \frac{3}{2}, \frac{t_4}{t_3} = \frac{4}{3}$

Since, $\frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$

∴ The given sequence is not a geometric progression.

2. For the G.P.

(i) if $r = \frac{1}{3}$, $a = 9$; find t_7

Ans. Given, $r = \frac{1}{3}$, $a = 9$

$t_n = ar^{n-1}$

$\therefore t_7 = 9 \times \left(\frac{1}{3}\right)^{7-1}$

$= \quad 9 \times \left(\frac{1}{3}\right)^6$

$= \quad 9 \times \frac{1}{729}$

$\therefore t_7 = \frac{1}{81}$

(ii) if $a = \frac{7}{243}$, $r = \frac{1}{3}$ find t_3

Ans. Given, $a = \frac{7}{243}$, $r = \frac{1}{3}$

$t_n = ar^{n-1}$

$\therefore t_3 = \frac{7}{243} \times \left(\frac{1}{3}\right)^{3-1}$

$= \frac{7}{243} \times \left(\frac{1}{3}\right)^2$

$= \frac{7}{243} \times \frac{1}{9}$

$= \frac{7}{2187}$

iii) if $a = 7$, $r = -3$ find t_6

Ans. Given, $a = 7$ $r = -3$

$t_n = ar^{n-1}$

$\therefore t_6 = 7 \times (-3)^{6-1}$

$= 7 \times (-3)^5$

$= 7 \times (-243)$

$= -1701.$

iv) if $a = \frac{2}{3}$, $t_6 = 162$, find r

Ans. Given, $a = \frac{2}{3}$ $t_6 = 162$

$t_n = ar^{n-1}$

$\therefore t_6 = \frac{2}{3} r^{6-1}$

$\therefore 162 = \frac{2}{3} r^5$

$\therefore r^5 = 162 \times \frac{3}{2}$

$\therefore r^5 = 3^5$

$\therefore r = 3.$

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3. Which term of the G.P. 5, 25, 125, 625, …….is 5^{10}?

Ans. Here, $t_1 = a = 5$, $r = \frac{t_2}{t_1} = \frac{25}{5} = 5$, $t_n = 5^{10}$

$t_n = ar^{n-1}$

$\therefore 5^{10} = 5 \times 5^{(n-1)}$

$\therefore 5^{10} = 5^{(1+n-1)}$

$\therefore 5^{10} = 5^n$

$\therefore n = 10$

$\therefore 5^{10}$ is the 10^{th} term of the G. P.

4. For what values of x.

$\frac{4}{3}, x, \frac{4}{27}$ are in G.P.?

Ans. $\frac{4}{3}, x, \frac{4}{27}$ are in geometric progression.

$\therefore \frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\therefore \frac{x}{\frac{4}{3}} = \frac{\frac{4}{27}}{x}$

$\therefore x^2 = \frac{4}{3}, \times, \frac{4}{27}$

$\therefore x^2 = \frac{16}{81}$

$\therefore x = \pm \frac{4}{9}$

5. If for a sequence, $t_n = \frac{5^{n-3}}{2^{n-3}}$, show that the sequence is a G.P.

Find its first term and the common ratio.

Ans. The sequence (t_n) is a G.P. if $\frac{t_{n+1}}{t_n}$ = constant for all $n \in N$.

Now, $t_n = \frac{5^{n-3}}{2^{n-3}}$

$\therefore t_{n+1} = \frac{5^{n+1-3}}{2^{n+1-3}} = \frac{5^{n-2}}{2^{n-2}}$

$\therefore \frac{t_{n+1}}{t_n} = \frac{5^{n-2}}{2^{n-2}} \times \frac{2^{n-3}}{2^{n-3}}$

$= (5)^{(n-2)-(n-3)} \times (2)^{(n-3)-(n-2)}$

$= (5)^1 \times (2)^{-1}$

$= \frac{5}{2}$

= constant, for all n ∈ N.

∴ The sequence is a G.P with common ratio (r) $\frac{5}{2}$

and first term = $t_1 = \frac{5^{1-3}}{2^{1-3}}$

$= \frac{5^{-2}}{2^{-2}}$

$= \frac{2^2}{5^2}$

$= \frac{4}{25}$

6. Find three numbers in G.P. such that their sum is 21 and sum of their squares is 189.

Ans. Let the three numbers in G.P. be $\frac{a}{r}$, a, ar.

According to the first condition.

$\frac{a}{r} + a + ar = 21$

$\therefore \frac{1}{r} + 1 + r = \frac{21}{a}$

$\therefore \frac{1}{r} + r = \frac{21}{a} - 1$ .... (i)

According to the second condition,

$\frac{a^2}{r^2} + a^2 + a^2r^2 = 189$

$\therefore \frac{1}{r^2} + 1 + r^2 = \frac{189}{a^2}$

$\therefore \frac{1}{r^2} + r^2 = \frac{189}{a^2} - 1$ ...(ii)

On squaring equation (i), we get

$\frac{1}{r^2} + r^2 + 2 = \frac{441}{a^2} - \frac{42}{a} + 1$

$\therefore \left(\frac{189}{a^2} - 1\right) + 2 = \frac{441}{a^2} - \frac{42}{a} + 1$ ... [From (ii)]

$\therefore \frac{189}{a^2} + 1 = \frac{441}{a^2} - \frac{42}{a} + 1$

$\therefore \frac{441}{a^2} - \frac{189}{a^2} - \frac{42}{a} = 0$

$\therefore \frac{252}{a^2} = \frac{42}{a}$

$\therefore 252 = 42a$

$\therefore a = 6$

Substituting the value of a in (i), we get

$\frac{1}{r} + r = \frac{21}{6} - 1$

$\therefore \frac{1 + r^2}{r} = \frac{15}{6}$

$\therefore \frac{1 + r^2}{r} = \frac{5}{2}$

$\therefore 2r^2 - 5r + 2 = 0$

$\therefore 2r^2 - 4r - r + 2 = 0$

$\therefore (2r - 1)(r - 2) = 0$

$\therefore r = \frac{1}{2}$ or 2

When a = 6, $r = \frac{1}{2}$

$\frac{a}{r} = 12, a = 6, ar = 3$

When a = 6, r = 2

$\frac{a}{r} = 3, a = 6, ar = 12$

$\therefore$ the three numbers are 12, 6 ,3 or 3,6, 12.

7. Find four numbers in G.P. such that sum of the middle two numbers is $\frac{10}{3}$ and their product is 1.

Ans. Let the four numbers in G.P be $\frac{a}{r^3}, \frac{a}{r}$, and ar, ar^3.

According to the second condition,

$\frac{a}{r^3}\left(\frac{a}{r}\right)(ar)(ar^3) = 1$

$\therefore a^4 = 1$

$\therefore a = 1$

According to the first condition,

$\frac{a}{r} + ar \frac{10}{3}$

$\therefore \frac{1}{r} + (1)\, r = \frac{10}{3}$

$\therefore \frac{1+r^2}{r} = \frac{10}{3}$

$\therefore 3 + 3r^2 = 10r$

$\therefore 3r^2 - 10r + 3 = 0$

$\therefore (r - 3)(3r - 1) = 0$

$\therefore r = 3$ or $r = \frac{1}{3}$

When r = 3, a = 1

$\frac{a}{r^3} = \frac{1}{(3)^3} = \frac{1}{27}$, $\frac{a}{r} = \frac{1}{3}$, ar = 1(3) = 3 and $ar^3 = 1(3)^3 = 27$

When r $\frac{1}{3}$, a = 1

$\frac{a}{r^3} = \frac{1}{(\frac{1}{3})^3} = 27$, $\frac{a}{r} = \frac{1}{(\frac{1}{3})} = 3$,

$ar = 1\left(\frac{1}{3}\right) = \frac{1}{3}$ and $ar^3 = 1\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

∴ the four numbers in G.P. are

$\frac{1}{27}, \frac{1}{3}$, 3, 27 or 27, 3, $\frac{1}{3}, \frac{1}{27}$.

8. Find five numbers in G. P. such that their product is 1024 and fifth term is square of the third term

Ans. Let the five numbers in G. P be

$\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2

According to the given conditions,

$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 1024$

$\frac{a}{\cancel{r^2}} \times \frac{a}{\cancel{r}} \times a \times a\cancel{r} \times a\cancel{r^2} = 1024$

∴ $a^5 = 4^5$

∴ a = 4 …(i)

Also, $ar^2 = a^2$

∴ $r^2 = \frac{a^2}{a}$

∴ $r^2 = a$

∴ $r^2 = 4$ …[From (i)]

∴ r = ± 2

When a = 4, r = 2

$\frac{a}{r^2} = 1$, $\frac{a}{r} = 2$, a = 4, ar = 8, $ar^2 = 16$

When a = 4, r = –2

$\frac{a}{r^2} = 1$, $\frac{a}{r} = -2$, a = 4, ar = –8, $ar^2 = 16$

The five numbers in G.P. are

1, 2, 4, 8, 16 or 1, –2, 4, –8,16.

9. The fifth term of a G.P. is x , eighth term of the G.P. is y and eleventh term of the G.P. is z Verify whether $y^2 = x\,z$.

Ans. Give, $t_5 = x$, $t_8 = y$, $t_{11} = z$

Since, $t_n = ar^{n-1}$

$\therefore t_5 = ar^4$, $t_8 = ar^7$, $t_{11} = ar^{10}$

consider,

L.H.S. $= y^2 = (t_8)^2 = (ar^7)^2 = a^2r^{14}$

R.H.S. $= xz = t_5, t_{11} = ar^4 , ar^{10} = a^2 r^{14}$

L.H.S. = R.H.S.

$y^2 = xz$

10. If p, q, r, s are in G.P. show that p + q , q + r , r + s are also in G.P

Ans. p, q, r, s are in G.P.

$\therefore \frac{q}{p} = \frac{r}{q} = \frac{s}{r}$

Let $\frac{q}{p} = \frac{r}{q} = \frac{s}{r} = k$

$\therefore q = pk, r = qk, s = rk$

We have to prove that p + q, q + r, r + s are in G.P.

i.e. to prove that $\frac{q+r}{p+q} = \frac{r+s}{q+r}$

L.H.S. $= \frac{q+r}{p+q} = \frac{q+qk}{p+pk} = \frac{q\,(1+k)}{p\,(1+k)} = \frac{q}{p} = k$

R.H.S. $= \frac{r+s}{q+r} = \frac{r+rk}{q+qk} = \frac{r\,(1+k)}{q\,(1+k)} = \frac{r}{q} = k$

$\therefore \frac{q+r}{p+q} = \frac{r+s}{q+r}$

$\therefore$ p + q, q + r, r + s are in G.P.

EXERCISE 4.2

1. For the following G.P.s, find S_n

i) 3 , 6 , 12 , 24 , ...

Ans. Here, a = 3, r = $\frac{6}{3}$ = 2 > 1

$S_n = \frac{a(r^n-1)}{r-1}$, for r > 1

$\therefore S_n = \frac{3(2^n-1)}{2-1}$

$\therefore S_n = 3(2^n-1)$

ii) p , q , $\frac{q^2}{p}$, $\frac{q^3}{q^2}$, ……..

Ans. Here, a = p, r = $\frac{q}{p}$

Let $\frac{q}{p} < 1$

$S_n = \frac{a(1-r^n)}{1-r}$, for r < 1

$S_n = \frac{p\left[1-\left(\frac{q}{p}\right)^n\right]}{1-\frac{q}{p}}$

$S_n = \frac{p^2}{p-q}\left[1-\left(\frac{q}{p}\right)^n\right]$

Let $\frac{q}{p} > 1$

$S_n = \frac{a(r^n-1)}{r-1}$, for r >1

$S_n = \frac{p\left[\left(\frac{q}{p}\right)^n-1\right]}{\frac{q}{p}-1}$

$= \frac{p^2}{q-p}\left[\left(\frac{q}{p}\right)^n - 1\right]$

2. For a G.P. if

i) a = 2, r = – $\frac{2}{3}$, find S_6

Ans. $S_n = \frac{a(1-r^n)}{1-r}$, for r < 1

$\therefore S_6 = \frac{2\left[1-\left(-\frac{2}{3}\right)^6\right]}{1-\left(-\frac{2}{3}\right)}$

$= \frac{2\left[1-\left(-\frac{2}{3}\right)^6\right]}{\frac{5}{3}}$

$= \frac{6}{5}\left[\frac{729-64}{3^6}\right]$

$= \frac{6}{5}\left[\frac{665}{729}\right]$

$\therefore S_6 = \frac{266}{243}$.

ii) $S_5 = 1023$, $r = 4$, Find a

Ans. $S_n = a\left(\frac{r^n-1}{r-1}\right)$, for $r > 1$

$\therefore S_5 = a\left(\frac{4^5-1}{4-1}\right)$

$\therefore 1023 = a\left(\frac{1024-1}{3}\right)$

$\therefore 1023 = \frac{a}{3}(1023)$

$\therefore a = 3.$

3. For a G.P. if

i) $a = 2$, $r = 3$, $S_n = 242$ find n.

Ans. $S_n = a\left(\frac{r^n-1}{r-1}\right)$, for $r > 1$

$\therefore 242 = 2\left(\frac{3^n-1}{3-1}\right)$

$\therefore 242 = 3^n - 1$

$\therefore 3^n = 243$

$\therefore 3^n = 3^5$

$\therefore n = 5.$

ii) sum of first 3 terms is 125 and sum of next 3 terms is 27, find the value of r.

Ans. $S_3 = 125$, $S_6 = 125 + 27 = 152$

$S_n = a\left(\frac{1-r^n}{1-r}\right)$

$\therefore S_3 = a\left(\frac{1-r^3}{1-r}\right)$

$\therefore 125 = a\left(\frac{1-r^3}{1-r}\right)$ … (i)

Also, $S_6 = a\left(\frac{1-r^6}{1-r}\right)$

$\therefore 152 = a\left(\frac{1-r^6}{1-r}\right)$ …(ii)

Dividing (ii) by (i), we get

$\frac{152}{125} = \frac{1-r^6}{1-r^3}$

$\therefore \frac{152}{125} = \frac{(1+r^3)(1-r^3)}{(1-r^3)}$

$\therefore 1 + r^3 = \frac{152}{125}$

$\therefore r^3 = \frac{152}{125} - 1$

$\therefore r^3 = \frac{27}{125}$

$\therefore r^3 = \left(\frac{3}{5}\right)^3$

$\therefore r = \frac{3}{5}$

4. For a G.P.

i) If $t_3 = 20$, $t_6 = 160$, find S_7

Ans. $t_3 = 20$, $t_6 = 160$

$t_n = ar^{n-1}$

$\therefore t_3 = ar^{3-1} = ar^2$

$\therefore ar^2 = 20$

$\therefore a = \frac{20}{r^2}$ …(i)

Also, $t_6 = ar^5$

$\therefore ar^5 = 160$

$\therefore \left(\frac{20}{r^2}\right) r^5 = 160$ …[From (i)]

$\therefore r^3 = \frac{160}{20} = 8$

$\therefore r = 2$

Substituting the value of r in (i), we get

$a = \frac{20}{2^2} = 5$

Now, $S_n = \frac{a(r^n-1)}{r-1}$, for > 1

$\therefore S_7 = \frac{5(2^7-1)}{2-1}$

$= 5(128 - 1)$

$= 635$

ii) If $t_4 = 16$, $t_9 = 512$, find S_{10}

Ans. $t_4 = 16, t_9 = 512$

$t_n = ar^{n-1}$

$\therefore t_4 = ar^{4-1} = ar^3$

$\therefore a = \frac{16}{r^3}$ …(i)

Also, $t_9 = ar^8$

$\therefore ar^8 = 512$

$\therefore \frac{16}{r^3} \times r^8 = 512$

$\therefore r^5 = 32$

$\therefore r = 2$

Substituting r = 2 in (i), we get

$a = \frac{16}{2^3} = \frac{16}{8} = 2$

Now, $S_n = \frac{a(r^n-1)}{r-1}$ for r > 1

$S_{10} = \frac{2(2^{10}-1)}{2-1}$

$= 2(1024 - 1)$

$= 2046$

5. Find the sum to n terms

i) 3 + 33 + 333 + 3333 + ...

Ans. $S_n = 3 + 33 + 333 + 3333 + \ldots$upto n terms

$= 3(1 + 11 + 111 + \ldots$ upto n terms)

$= \frac{3}{9}(9 + 99 + 999 + \ldots$upto n terms)

$= \frac{1}{3}[(10 - 1) + (100 - 1) + (1000 - 1) + \ldots$ upto n terms)

$= \frac{1}{3}$ [(10 + 100 + 1000 + … upto n terms) – (1 + 1 + 1 + … n terms)

But 10, 100, 1000, … n terms are in G.P.

With a = 10, r = $\frac{100}{10}$ = 10

$\therefore S_n = \frac{1}{3}\left[10\left(\frac{10^n - 1}{10-1}\right) - n\right]$

$\therefore S_n = \frac{1}{3}\left[\frac{10}{9}(10^n - 1) - n\right]$

ii) 8 +88 + 888 + 8888 + ...

Ans. S_n = 8 +88 + 888 + ... upto n terms

= 8 (1 + 11+ 111 + … upto n terms)

$= \frac{8}{9}$(9 + 99 + 999 + …upto n terms)

$= \frac{8}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) + … upto n terms)

$= \frac{8}{9}$ [(10 + 100 + 1000 + … upto terms) – (1 + 1 + 1 + …n terms)

But 10, 100, 1000, … n terms are in G.P. With

a = 10, r = $\frac{100}{10}$ = 10

$\therefore S_n = \frac{8}{9}\left[10\left(\frac{10^n - 1}{10-1}\right) - n\right]$

$= \frac{8}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

$\therefore S_n = \frac{8}{81}[10(10^n - 1) - 9n]$

6. Find the sum to n terms

i) 0.4 + 0.44 + 0.444 + ...

Ans. S_n = 0.4 + 0.444 + 0.444 + ... upto n terms

$S_n = \frac{4}{10} + \frac{44}{100} + \frac{444}{100}$ + n terms

$S_n = 4\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \cdots \text{n term}\right]$

$\frac{4}{9} = \left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \cdots \text{n term}\right]$

$S_n = \frac{4}{9}\left[1 - \frac{1}{10} + 1 - \frac{1}{100} + 1 - \frac{1}{1000} + \cdots \text{n term}\right]$

$S_n = \frac{4}{9}\left[1+1+1+\cdots \text{n term} - \left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\cdots \text{n term}\right)\right]$

Here, $a = \frac{1}{10}$ r, $= \frac{1}{10}$, $S_n = \frac{a(1-r^n)}{1-r}$

$S_n = \frac{4}{9}\left[n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$

$= \frac{4}{9}\left[n - \frac{1}{10} \times \frac{10}{9}\left(1-\frac{1}{10^n}\right)\right]$

$S_n = \frac{4}{9} n = \left[\frac{4}{81}\left(1-\frac{1}{10^n}\right)\right]$

ii) 0.7 + 0.77+ 0.777 + ...

Ans. S_n = 0.7 + 0.77 + 0.777 + ... upto n terms

= 7 (0.1 + 0.11+ 0.111 + … upto n terms)

$\frac{7}{9}$ = (0.9 + 0.99 + 0.999 + upto n terms)

$= \frac{7}{9}$ [(1 – 0.1) + (1 – 0.01) + (1 – 0.001) + upto n terms]

$\frac{7}{9}$ [1 + 1 + 1 ... n times) – (0.01 + 0.01 + 0.001 + ⋯ upto n terms)]

But 0.1, 0.01, 0.001 … n terms are in G.P.

With a = 0.1, $r = \frac{0.01}{0.1} = 0.01$

$S_n = \frac{7}{9}\left\{n - 0.1\left[\frac{1-(0.1)^n}{1-0.1}\right]\right\}$

$S_n = \frac{7}{9}\left\{n - \frac{0.1}{0.9}[1-(0.1)^n]\right\}$

$S_n = \frac{7}{9}\left[n - \frac{1}{9}[1-(0.1)^n]\right]$

$S_n = \frac{7}{81}\left\{9n - \left(1-\frac{1}{10^n}\right)\right\}$

7. Find the n^{th} term of the sequence

i) 0.5, 0.55, 0.555, ...

Ans. Let t_1 = 0.5, t_2 = 0.55, t_3 = 0.555 and so on

t_1 =0.5

t_2 = 0.55 = 0.5 + 0.05

t_3 = 0.555 = 0.5 + 0.05 + 0.005

∴ t_n = 0.5 + 0.05 + 0.005 + … upto n terms

But 0.5 , 0.05 , 0.005 + … upto n terms are in

G.P. with a = 0.5 and r = 0.1

t_n = the sum of first n terms of the G.P

$\therefore t_n = 0.5 \left\{\frac{1-(0.1)^n}{1-0.1}\right\}$

$\therefore t_n = \frac{0.5}{0.9} \{1 - (0.1)^n\}$

$\therefore t_n = \frac{5}{9} \{1 - (0.1)^n\}$

ii) 0.2, 0.22, 0.222, ...

Ans. Let $t_1 = 0.2$, $t_2 = 0.22$, $t_3 = 0.222$ and so on

$t_1 = 0.2$

$t_2 = 0.22 = 0.2 + 0.02$

$t_3 = 0.222 = 0.2 + 0.02 + 0.002$

$\therefore t_n = 0.2 + 0.02 + 0.002 + \ldots$ upto n terms

But 0.2 , 0.02, 0.002 + … upto n terms are in

G.P. with a = 0.2 and r = 0.1

$\therefore t_n$ = the sum of first n terms of the G.P

$\therefore t_n = 0.2 \left\{\frac{1-(0.1)^n}{1-0.1}\right\}$

$\therefore t_n = \frac{0.2}{0.9} \{1 - (0.1)^n\}$

$\therefore t_n = \frac{2}{9} \{1 - (0.1)^n\}$

8. For a sequence, if $S_n = 2 (3^n -1)$, find the n^{th} term , hence show that the sequence is a G.P.

Ans. $S_n = 2(3^n - 1)$

$\therefore S_{n-1} = 2 (3^n - 1 - 1)$

But $t_n = S_n - S_{n-1}$

$= 2 (3^n - 1) - 2 (3^{n-1} - 1)$

$= 2 (3^n - 1 - 3^{n-1} + 1)$

$= 2 (3^n - 3^{n-1})$

$= 2 (3^{n-1+1} - 3n^{-1})$

$\therefore t_n \; 2.3^{n-1} (3 - 1) = 4. 3^{n-1}$

$\therefore t_{n+1} = 4.3^{(n+1-1)}$

$= 4 (3)^n$

The sequence (t_n) is a G.P. if $\frac{t_{n+1}}{t_n}$ = constant for all n ϵ N.

$\frac{t_{n+1}}{t_n} = \frac{4\,(3)^n}{4\,(3)^{n-1}}$

= 3 = constant, for all n ϵ N.

∴ the sequence is a G.P. with $t_n = 4\,(3)^{n-1}$.

9. If S,P,R are the sum , product and sum of the reciprocals of n terms of a G. P. respectively, then verify that $\left(\frac{S}{R}\right)^n = P^2$.

Ans. Let a be the 1st term and r b the common ratio of the G.P.

∴ the G.P. is a, ar, ar^2 +, ar^3, … ar^{n-1}

∴ $S = a + ar + ar^2 + \ldots + ar^{n-1} = a\left(\frac{r^n - 1}{r-1}\right)$

$P = a\,(ar)\,(ar)^2 \ldots (ar^{n-1})$

$= a^n \cdot r^{1+2+3\ldots+(n-1)}$

$= a^n \cdot r^{\frac{n(n-1)}{2}}$

∴ $P = a^{2n} \cdot r^{n(n-1)}$ (i)

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \ldots + \frac{1}{ar^{n-1}}$

$= \frac{r^{n-1} + r^{n-2} + r^{n-3} + \cdots + r^2 + r + 1}{a.r^{n-1}}$

$= \frac{1 + r + r^2 + \ldots + r^{n-2} + r^{n-1}}{a.r^{n-1}}$

$1, r, r^2, \ldots, r^{n-1}$ are in G.P., with a = 1, r = r

∴ $R = \frac{1}{ar^{n-1}}\left(\frac{r^n - 1}{r-1}\right) = \frac{1}{a^2 \cdot r^{n-1}} \times a \times \left(\frac{r^n - 1}{r-1}\right)$

∴ $R = \frac{1}{a^2 \cdot r^{n-1}} S$

∴ $a^2 \cdot r^{n-1} = \frac{S}{R}$

∴ $(a^2 \cdot r^{n-1})^n = \left(\frac{S}{R}\right)^n$

∴ $a^{2n} \cdot r^{n(n-1)} = \left(\frac{S}{R}\right)^n$

∴ $P^2 = \left(\frac{S}{R}\right)^n$ …[From (i)]

10. If S_n ,S_{2n} ,S_{3n} are the sum of n,2n,3n terms of a G.P. respectively , then verify that $S_n (S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$.

Ans. Let a and r be the 1st term and common ratio of the G.P. respectively.

$S_n = a\left(\frac{r^n-1}{r-1}\right)$, $S_{2n} = a\left(\frac{r^{2n}-1}{r-1}\right)$, $S_{3n} = a\left(\frac{r^{3n}-1}{r-1}\right)$

$\therefore S_{2n} - S_n = a\left(\frac{r^{2n}-1}{r-1}\right) - a\left(\frac{r^n-1}{r-1}\right)$

$= \frac{a}{r-1}(r^{2n} - 1 - r^n + 1)$

$= \frac{a}{r-1}(r^{2n} - r^n)$

$= a\frac{r^n}{r-1}(r^n - 1)$

$\therefore S_{2n} - S_n = r^n \cdot \frac{a(r^n-1)}{r-1}$ …(i)

$S_{3n} - S_{2n} = a\left(\frac{r^{3n}-1}{r-1}\right) - a\left(\frac{r^{2n}-1}{r-1}\right)$

$= \frac{a}{r-1}(r^{3n} - 1 - r^{2n} + 1)$

$= \frac{a}{r-1}(r^{3n} - r^{2n})$

$= \frac{a}{r-1} \cdot r^{2n}(r^n - 1)$

$= a.\left(\frac{r^n-1}{r-1}\right) \cdot r^{2n}$

$\therefore S_n (S_{3n} - S_{2n}) = \left[a.\left(\frac{r^n-1}{r-1}\right)\right]\left[a.\left(\frac{r^n-1}{r-1}\right)r^{2n}\right]$

$= \left[r^n . \frac{a(r^n-1)}{r-1}\right]^2$

$\therefore S_n (S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$ … [From (i)]

EXERCISE 4.3

1. Determine whether the sums to infinity of the following G.P.s exist ,if exist find them

i) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\ldots\ldots\ldots$

Ans. Here, $a = \frac{1}{2}$, $r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

Since, $|r| = |\frac{1}{2}| < 1$

∴ Sum to infinity exists

Sum to infinity $= \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1.$

ii) $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \ldots\ldots\ldots\ldots$

Ans. $a = 2$, $r = \frac{\frac{4}{3}}{2} = \frac{2}{3}$

Since, $|r| = |\frac{2}{3}| < 1$

∴ Sum to infinity exists.

Sum to infinity $= \frac{a}{1-r} = \frac{2}{1-\frac{2}{3}} = 6.$

iii) $-3, 1, \frac{-1}{3}, \frac{1}{9}, \ldots\ldots\ldots\ldots$

Ans. $a = -3$, $r = -\frac{1}{3}$

Since, $|r| = |-\frac{1}{3}| < 1$

∴ Sum to infinity exists.

Sum to infinity $= \frac{a}{1-r} = \frac{-3}{1-\left(-\frac{1}{3}\right)} = \frac{-9}{4}$

iv) $\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5}$ ……..

Ans. $a = \frac{1}{5}$, $r = \frac{\frac{-2}{5}}{\frac{1}{5}} = -2$

Since, $|r| = |-2| < 1$

∴ Sum to infinity does not exists.

2. Express the following recurring decimals as a rational number.

i) $0.\overline{32}$.

Ans. $0.\overline{32} = 0.323232\ldots$

$= 0.32 + 0.0032 + 0.000032 + \ldots$

Here, 0.32, 0.0032, 0.000032, … are in G.P. with a = 0.32 and r = 0.01

Since, $|r| = |0.01| < 1$

∴ Sum to infinity exists.

∴ Sum to infinity $= \frac{a}{1-r}$

∴ $0.\overline{32} = \frac{0.32}{1-(0.01)} = \frac{0.32}{0.99}$

∴ $0.\overline{32} = \frac{32}{99}$

ii) $3.\dot{5}$

Ans. 3.555 …

$= 3 + 0.5 + 0.05 + 0.005 + \ldots.$

Here, 0.5, 0.05, 0.005, … are in G.P. with a = 0.5 and r = 0.1.

Since $|r| = |0.1| < 1$

∴ Sum to infinity exists.

∴ Sum to infinity $= \frac{a}{1-r}$

$= \frac{0.5}{1-(0.1)}$

$= \frac{0.5}{0.9}$

$= \frac{5}{9}$

∴ $3.5 = 3 + \frac{5}{9} = \frac{32}{9}$

iii) $4.\overline{18}$

Ans. $4.\overline{18} = 4.181818\ldots$

$= 4 + 0.18 + 0.0018 + 0.000018 + \ldots$

Here, 0.18 0.0018, 0.000018, … are in G.P.

With a = 0.18 and r = 0.01

Since, $|r| = |0.01| < 1$

∴ Sum to infinity exists.

$\therefore$ Sum to infinity $= \frac{a}{1-r} = \frac{0.18}{1-(0.01)} = \frac{0.18}{0.99} = \frac{18}{99} = \frac{2}{11}$

$= 4.\overline{18} = 4 + \frac{2}{11} = \frac{46}{11}$.

iv) $0.3\overline{45}$

Ans. $= 0.3 + 0.045 + 0.00045 + 0.0000045 +$

Here, 0.045, 0.00045 + 0.00000045 , … are in

G.P. with a = 0.045, r = 0.01

Since, | r | = |0.01| < 1

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity $= \frac{a}{1-r} = \frac{0.045}{1-0.01}$

$= \frac{0.045}{0.99} = \frac{45}{990}$

$\therefore 0.3\overline{45} = 0.3 + \frac{45}{990}$

$= \frac{3}{10} + \frac{1}{22}$

$= \frac{33+5}{110}$

$= \frac{38}{110}$

$= \frac{19}{55}$

Alternate method:

$0.3\overline{45} = \frac{3\overline{45}}{10}$

$= \frac{3+0.45+0.0045+0.000045+\cdots}{10}$

Here, 0.45, 0.0045, 0.000045…. are in G.P.

With a = 0.45 and r = 0.01

Since, | r | = |0.01 | < 1

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity $\frac{a}{1-r} = \frac{0.45}{1-0.01}$

$= \frac{0.45}{0.99} = \frac{45}{99} = \frac{5}{11}$

$\therefore 0.3\overline{45} = \frac{3+\frac{5}{11}}{10} = \frac{\frac{38}{11}}{10} = \frac{19}{55}$.

v) $3.4\overline{56}$

Ans. $3.4\overline{56} = 3.4565656\ldots$

$= 3.4 + 0.056 + 0.00056 + 0.0000056 + \ldots$

Here, 0.056, 00056, 0.0000056, … are in

G.P. with a = 0.056 and r = 0.01

Since $|r| = |0.01| < 1$

∴ Sum to infinity exists

∴ Sum to infinity $= \frac{a}{1-r}$

$= \frac{0.056}{1-0.01}$

$= \frac{0.056}{0.99}$

$= \frac{56}{990}$

$\therefore 3.4\overline{56} = 3.4 + \frac{56}{990}$

$= \frac{34}{10} + \frac{56}{990}$

$= \frac{3366+56}{990}$

$= \frac{3422}{990}$

$= \frac{1711}{495}$.

3. If the common ratio of a G.P. is $\frac{2}{3}$ and sum of its terms to infinity is 12. Find the first term.

Ans. $r = \frac{2}{3}$ sum to infinity = 12 …[Given]

Sum to infinity $\frac{a}{1-r}$

$\therefore 12 = \frac{a}{1-\frac{2}{3}}$

$\therefore a = 12 \times \frac{1}{3}$

∴ a = 4.

4. If the first term of a G.P. is 16 and sum of its terms to infinity is $\frac{176}{5}$, find the common ratio

Ans. a = 16, sum to infinity $= \frac{176}{5}$ …[Given]

Sum to infinity $\frac{a}{1-r}$

$\therefore \frac{176}{5} = \frac{16}{1-r}$

$\therefore \frac{11}{5} = \frac{1}{1-r}$

$\therefore 11 - 11r = 5$

$\therefore 11r = 6$

$\therefore r = \frac{6}{11}$.

5. The sum of the terms of an infinite G.P. is 5 and the sum of the squares of those terms is 15. Find the G.P.

Ans. Let the required G.P. be a, ar, ar^2, ar^3, …

sum to infinity of this G.P. = 5

$\therefore 5 = \frac{a}{1-r}$

$\therefore a = 5(1 - r)$ …(i)

Also, the sum of the squares of the terms is 15.

$\therefore (a^2 + a^2r^2 + a^2r^4 + \dots) = 15$

$\therefore 15 = \frac{a^2}{1-r^2}$

$\therefore 15(1 - r^2) = a^2$

$\therefore 15(1 - r)(1 + r) = 25(1 - r)^2$ …[From (i)]

$\therefore 3(1 + r) = 5(1 - r)$

$\therefore 3 + 3r = 5 - 5r$

$\therefore 8r = 2$

$\therefore r = \frac{1}{4}$

$\therefore a = 5\left(1 - \frac{1}{4}\right) = 5\left(\frac{3}{4}\right) = \frac{15}{4}$

$\therefore$ Required G.P. is a, ar, ar^2, ar^3, …

i.e., $\frac{15}{4}, \frac{15}{16}, \frac{15}{64}, \dots$.

EXERCISE 4.4

1. Verify whether the following sequences are H.P.

i) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots\ldots\ldots$

Ans. Here, the reciprocal sequence is 3, 5, 7, 9 ….

$\therefore t_1 = 3, t_1 = 5, t_3 = 7, \ldots\ldots$

$\because t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 2$, constant

$\therefore$ The reciprocal sequence is an A.P.

$\therefore$ the given sequence is H.P.

ii) $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \ldots\ldots\ldots$

Ans. Here, the reciprocal sequence is 3, 6, 9, 12 ….

$\therefore t_1 = 3, t_2 = 6, t_3 = 9, t_4 = 12 \ldots\ldots$

$\because t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 3$, constant

$\therefore$ The reciprocal sequence is an A.P.

$\therefore$ the given sequence is H.P.

iii) $\frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \frac{1}{15} \ldots\ldots\ldots$

Ans. Here, the reciprocal sequence is

7, 9, 11, 13, 15, …

$\therefore t_1 = 7, t_2 = 9, t_3 = 11, t_4 = 13 \ldots\ldots$

$\because t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 2$, constant

$\therefore$ The reciprocal sequence is an A.P.

$\therefore$ the given sequence is H.P.

2. Find the n^{th} term and hence find the 8^{th} term of the following H.P.s

i) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \ldots\ldots$

Ans. $\therefore$ 2, 5, ,8, 11, ….. are in A.P.

$\therefore a = 2, d = 3$

$t_n = a + (n - 1)\, d$

$= 2 + (n - 1)\,(3)$

$= 3n - 1$

$\therefore$ n^{th} term of H.P. is $\frac{1}{3n-1}$

$\therefore$ 8^{th} term of H.P. is $= \frac{1}{3(8)-1} = \frac{1}{23}$

ii) $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots\ldots$

Ans. $\therefore$ 4, 6, 8, 10 … are in A.P.

$\therefore a = 4, d = 2$

$t_n = a + (n - 1)\, d$

$= 4 + (n - 1)\,(2)$

$= 2n + 2$

$\therefore$ n^{th} term of H.P. $= \frac{1}{2n + 2}$

$\therefore$ 8^{th} term of H.P. $= \frac{1}{2\,(8) + 2} = \frac{1}{18}$

iii) $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \ldots\ldots$

Ans. 5, 10, 15, 20, … are in H.P.

$\therefore a = 5, d = 5$

$\therefore t_n = a + (n - 1)d$

$= 5 + (n - 1)\,(5)$

$= 5n$

$\therefore$ n^{th} term of H.P. $= \frac{1}{5n}$

$\therefore$ 8^{th} term of H.P. $= \frac{1}{5\,(8)} = \frac{1}{40}$.

3. Find A.M. of two positive numbers whose G.M. and H. M. are 4 and $\frac{16}{5}$.

Ans. G.M. = 4, H.M. $= \frac{16}{5}$

$\because (G.M.)^2 = (A.\ M.)\ (H.M.)$

$\therefore 16 = A.M. \times \frac{16}{5}$

$\therefore A.M = 5$

4. Find H.M. of two positive numbers whose A.M. and G.M. are $\frac{15}{2}$ and 6

Ans. A. M. $\frac{15}{2}$, G.M. = 6

Now, $(G.M.)^2 = (A.\ M.)\ (H.M.)$

$\therefore 6^2 = \frac{15}{5} \times H.\ M.$

$\therefore H.M = 36 \times \frac{2}{15}$

$\therefore H.M = \frac{24}{5}$

5. Find G.M. of two positive numbers whose A.M. and H.M. are 75 and 48

Ans. A. M. = 75, H.M. = 48

$\therefore\ (G.M.)^2 = (A.\ M.)\ (H.M.)$

$\therefore\ (G.M.)^2 = 75 \times 48$

$\therefore 25 \times 3 \times 16 \times 3$

$\therefore 5^2 \times 4^2 \times 3^2$

$\therefore G.M. = 5 \times 4 \times 3$

$\therefore G.\ M. = 60$

6. Insert two numbers between $\frac{1}{7}$ and $\frac{1}{13}$ so that the resulting sequence is a H.P.

Ans. Let the required number be $\frac{1}{H_1}$ and $\frac{1}{H_2}$.

$\therefore \frac{1}{7}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{13}$ are in H.P.

$\therefore 7, H_1, H_2$ and 13 are in A.P.

$\therefore t_1 = a = 7$ and $t_4 = a + 3d = 13$

$\therefore 7 + 3d = 13$

$\therefore 3d = 6$

$\therefore\ d = 2$

$\therefore H_1 = t_2 = a + d = 7 + 2 = 9$

and $H_2 = t_3 = a + 2d = 7 + 2(2) = 11$

$\therefore \frac{1}{9}$ and $\frac{1}{11}$ are the required numbers to be inserted between $\frac{1}{7}$ and $\frac{1}{13}$ so that the resulting sequence is a H.P.

7. Insert two numbers between 1 and −27 so that the resulting sequence is a G.P.

Ans. Let the required number be G_1 and G_2

$\therefore$ 1, G_1, G_2,– 27 are in G.P.

$\therefore t_1 = 1, t_2 = G_1, t_3 = G_2, t_4 = -27$

$\therefore t_1 = a = 1$

$t_n = ar^{n-1}$

$\therefore t_4 = (1)r^{4-1}$

$\therefore -27 = r^3$

$\therefore r^3 = (-3)^3$

$\therefore r = -3$

$\therefore G_1 = t_2 = ar = 1(-3) = -3$

$G_2 = t_3 = ar^2 = 1(-3)^2 = 9$

$\therefore$ – 3 and 9 are the required numbers to be inserted between 1 and –27 so that the resulting sequence is a G.P.

8. Find two numbers whose A.M. exceeds their G.M. by $\frac{1}{2}$ and their H.M. by $\frac{25}{26}$

Ans. Let a, b be the two numbers.

$A = \frac{a+b}{2}$, $G = \sqrt{ab}$ H, $= \frac{2ab}{a+b}$,

According to the given conditions,

$A = G + \frac{1}{2}$, $A = H + \frac{25}{26}$

$\therefore G = A - \frac{1}{2}$, $H = A - \frac{25}{26}$ …(i)

Now, $G^2 = AH$

$\left(A - \frac{1}{2}\right)^2 = A\left(A - \frac{25}{26}\right)$

$\therefore A^2 - A + \frac{1}{4} = A^2 - \frac{25}{26}A$

$\therefore A - \frac{25}{26}A = \frac{1}{4}$

$\therefore \frac{1}{26}A = \frac{1}{4}$

$\therefore A = \frac{13}{2}$ …(ii)

$\therefore G = 6$ …[From (i) and (ii)]

$\therefore \quad \frac{a+b}{2} = \frac{13}{2}$ and $\sqrt{ab} = 6$

$\therefore \quad a + b = 13,$

$\therefore \quad b = 13 - a \qquad \ldots$ (iii)

and $ab = 36$

$\therefore a(13 - a) = 36 \qquad \ldots$ [From (iii)]

$\therefore a^2 - 13a + 36 = 0$

$\therefore \quad (a - 4)(a - 9) = 0$

$\therefore \quad a = 4$ or $a = 9$

When $a = 4$, $b = 13 - 4 = 9$

When $a = 9$, $b = 13 - 9 = 4$

$\therefore$ the two numbers are 4 and 9.

9. Find two numbers whose A.M. exceeds G.M. by 7 and their H.M. by $\frac{63}{5}$.

Ans. Let a, b be the two numbers.

$A = \frac{a+b}{2}$, $G = \sqrt{ab}$ $H, = \frac{2ab}{a+b}$,

According to the given conditions,

$A = G + 7, A = H + \frac{63}{5}$

$\therefore G = A - 7 \qquad \ldots$(i)

$H = A - \frac{63}{5}$

Now, $G^2 = AH$

$\therefore \quad (A - 7)^2 = A\left(A - \frac{63}{5}\right)$

$\therefore A^2 - 14A + 49 = A^2 - \frac{63A}{5}$

$\therefore 14A - \frac{63A}{5} = 49$

$\therefore \frac{7A}{5} = 49$

$\therefore \quad A = 35$

$\therefore \quad \frac{a+b}{2} = 35$

$\therefore \quad a + b = 70,$

$\therefore \quad b = 70 - a \qquad \ldots$ (ii)

G = A –7 …[From (i)]

= 35 – 7

∴ G = 28

∴ $\sqrt{ab} = 28$

∴ ab = 28^2 = 784

∴ a (70 – a) = 784 …[From (ii)]

∴ 70a – a^2 = 784

∴ a^2 – 70a + 784

∴ a^2 – 56a – 14a + 784 = 0

∴ (a – 56) (a – 14) = 0

∴ a = 14 or a = 56

When a = 14, b = 70 – 14 = 14

When a = 56, b = 70 – 56 = 14

∴ the two numbers are 14 and 56.

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EXERCISE 4.5

1. Find the sum $\sum_{r=1}^{n}(r+1)(2r-1)$

Ans. $= \sum_{r=1}^{n}(2r^2 + r - 1)$

$= 2\ \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$

$= 2.\frac{n\,(n+1)(2n+1)}{6} + \frac{n\,(n+1)}{2} - n$

$= \frac{n}{6}[2\,(2n^2 + 3n + 1) + 3\,(n + 1) - 6]$

$= \frac{n}{6}\,(4n^2 + 6n + 2 + 3n + 3 - 6)$

$= \frac{n}{6}\,(4n^2 + 9n - 1)$

2. Find $\sum_{r=1}^{n}(3r^2 - 2r + 1)$

Ans. $3\sum_{r=1}^{n} r^2 - 2\ 2\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$

$= 3.\frac{n\,(n+1)(2n+1)}{6} - 2\frac{n\,(n+1)}{2} + n$

$= \frac{n}{2}[2n^2 + 3n + 1) - 2\,(n + 1) + 2]$

$= \frac{n}{2}\,(2n^2 + 3n + 1 - 2n - 2 + 2)$

$= \frac{n}{2}\,(2n^2 + n + 1)$

3. Find $\sum_{r=1}^{n}\frac{1+2+3+\cdots+r}{r}$

Ans. $\sum_{r=1}^{n}\frac{r\,(r+1)}{2r}$

$= \frac{1}{2}\sum_{r=1}^{n} r\,(r+1)$

$= \frac{1}{2}[\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1]$

$= \frac{1}{2}\left[\frac{n\,(n+1)}{2} + n\right]$

$= \frac{n}{4}[(n + 1) + 2]$

$= \frac{n}{4}\,(n + 3)$

4. Find $\sum_{r=1}^{n} \frac{1^3+2^3+\ldots+r^3}{r\,(r+1)}$

Ans. We know that,

$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2\,(n+1)^2}{4}$

$\therefore 1^3 + 2^3 + 3^3 + \ldots + r^3 = \frac{r^2\,(r+1)^2}{4}$

$\therefore \frac{1^3+2^3+3^3\ldots+r^3}{r\,(r+1)} = \frac{r\,(r+1)}{4}$

$\therefore \sum_{r=1}^{n} \left[\frac{1^3+2^3+3^3\ldots+r^3}{r\,(r+1)}\right]$

$= \sum_{r=1}^{n} \frac{r\,(r+1)}{4}$

$= \frac{1}{4} \sum_{r=1}^{n} (r^2 + r)$

$= \frac{1}{4} \left(\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right)$

$= \frac{1}{4}\left[\frac{n\,(n+1)(2n+1)}{6} + \frac{n\,(n+1)}{2}\right]$

$= \frac{1}{4}\frac{n\,(n+1)}{2}\left(\frac{2n+1}{3} + 1\right)$

$= \frac{n\,(n+1)}{8}\left(\frac{2n+1+3}{3}\right)$

$= \frac{n\,(n+1)\,(2n+4)}{24}$

$= \frac{2n\,(n+1)\,(n+2)}{24}$

$= \frac{n\,(n+1)\,(n+2)}{12}$

5. Find the sum 5 × 7 + 9 × 11 + 11 × 13 + --- upto n terms.

Ans. Now 5, 7, 9, 11, … are in A.P.

r^{th} term = 5 + (r–1) (2) = 2r + 3

7, 9, 11, … are in A.P.

r^{th} term = 7 + (r–1) (2) = 2r + 5

$\therefore$ 5 × 7 + 7 × 9 + 9 × 11 × 13 + --- upto n terms.

$= \sum_{r=1}^{n} (2r + 3)(2r + 5)$

$= \sum_{r=1}^{n} 4r^2 + 16r + 15$

$= 4 \sum_{r=1}^{n} r^2 + 16r \sum_{r=1}^{n} r + 15 \sum_{r=1}^{n} 1$

$= 4\left[\frac{n(n+1)(2n+1)}{6}\right] + 16\left[\frac{n(n+1)}{2}\right] 15n$

$= \frac{n}{3}[2(2n^2 + 3n + 1) + 24(n + 1) + 45]$

$= \frac{n}{3}(4n^2 + 6n + 2 + 24n + 24 + 45)$

$= \frac{n}{3}(4n^2 + 30n + 71)$

6. Find the sum $2^2 + 4^2 + 6^2 + 8^2 +$ --- upto n terms

Ans. $2^2 + 4^2 + 6^2 + 8^2 +$ --- upto n terms

$(2 \times 1)^2 + (2 \times 2)^2 + (2 \times 3)^2 + (2 \times 4)^2 + \ldots$

$= \sum_{r=1}^{n}(2r^2)$

$= 4\sum_{r=1}^{n} r^2$

$= \frac{4.n(n+1)(2n+1)}{6}$

$= \frac{2n(n+1)(2n+1)}{3}$.

7. Find $(70^2 - 69^2) + (68^2 - 67^2) + (66^2 - 65^2) +$ --- $+ (2^2 - 1^2)$

Ans. Let $s = (70^2 - 69^2) + (68^2 - 67^2) +$ --- $+ (2^2 - 1^2)$

$S = (2^2 - 1^2) + (4^2 - 3^2) + \ldots + (70^2 - 69^2)$

Here 2, 4, 6, …, 70 is an A.P. with r^{th} term = 2r

and 1, 3, 5 …, 69 in A.P. with r^{th} term = 2r – 1

$\therefore S = \sum_{r=1}^{35}(2r^2) - (2r - 1)^2]$

$= \sum_{r=1}^{35}[4r^2 - (4r^2 - 4r + 1)]$

$= \sum_{r=1}^{35}(4r - 1)$

$= 4\sum_{r=1}^{35} r - \sum_{r=1}^{35} 1$

$= 4.\frac{35 \times 36}{2} - 35$

$= (72 - 1)(35)$

$= 71 \times 35 = 2485.$

8. Find the sum $1\times 3 \times 5 + 3 \times 5 \times 7 + 5 \times 7 \times 9 +$ --- $(2n-1)(2n+1)(2n+3)$

Ans. Now, 1, 3, 5, 7, … are in A.P. with a = 1 and d = 2

$\therefore r^{th}$ term $= 1 + (r - 1)2 = 2r - 1$

3, 5, 7, 9, … are in A.P. with a = 3 and d = 2

$\therefore$ r^{th} term = 3 + (r – 1)2 = 2r + 1

and 5, 7, 9, 11, … are in A.P. with a = 5 and d = 2.

r^{th} term = 5 + (r–1) 2 = 2r + 3

$\therefore$ $1 \times 3 \times 5 + 3 \times 5 \times 7 + 5 \times 7 \times 9 +$ …upto n terms

$= \sum_{r=1}^{n} (2r-1)(2r+1)(2r+3)$

$= \sum_{r=1}^{n} (4r^2 - 1)(2r + 3)$

$= \sum_{r=1}^{n} (8r^3 + 12r^2 - 2r - 3)$

$= 8\sum_{r=1}^{n} r^3 + 12\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r - 3\sum_{r=1}^{n} 1$

$= 8\left\{\frac{n(n+1)}{2}\right\}^2 + 12\left\{\frac{n(n+1)(2n+1)}{6}\right\} - 2\left\{\frac{n(n+1)}{2}\right\} - 3n$

$= 2n^2(n+1)^2 + 2n(n+1)(2n+1) - n(n+1) - 3n$

$= n(n+1)[2n(n+1) + 4n + 2 - 1] - 3n$

$= n(n+1)(2n^2 + 6n + 1) - 3n$

$= n(2n^3 + 8n^2 + 7n + 1 - 3)$

$= n(2n^3 + 8n^2 + 7n - 2)$

9. Find n, if $\frac{1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \cdots + \text{upto n terms}}{1 + 2 + 3 + 4 + \cdots + \text{upto n terms}} = \frac{100}{3}$.

Ans. $\frac{\sum_{r=1}^{n} r(r+1)}{\sum_{r=1}^{n} r} \frac{100}{3}$

$\therefore$ $\frac{\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r}{\sum_{r=1}^{n} r} \frac{100}{3}$

$\therefore$ $\frac{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{\frac{n(n+1)}{2}} = \frac{100}{3}$

$\therefore$ $\frac{\frac{n(n+1)}{6}[(2n+1)+3]}{\frac{n(n+1)}{2}} = \frac{100}{3}$

$\therefore$ $\frac{2(n+2)}{3} = \frac{100}{3}$

$\therefore$ n + 2 = 50

$\therefore$ n = 48

10. If S_1 , S_2 and S_3 are the sums of first n natural numbers, their squares and their cubes respectively then show that $9\ S_2^2 = S_3\ (1+ 8\ S_1\ )$

Ans. $S_1 = 1 + 2 + 3 + \ldots + n = \sum_{r=1}^{n} \frac{n\ (n+1)}{2}$

$S_2 = 1^2 + 2^2 + 3^2 + \ldots n^2 = \sum_{r=1}^{n} \frac{n\ (n+1)(2n+1)}{6}$

$S_3 = 1^3 + 2^3 + 3^3 + \ldots n^3 = \sum_{r=1}^{n} r^3 \quad \frac{n^2\ (n+1)^2}{4}$

R. H. S. $= S_3\ (1 + 8S_1)$

$= \frac{n^2\ (n+1)^2}{4}\left[1 + 8.\frac{n\ (n+1)}{4}\right]$

$= \frac{n^2\ (n+1)^2}{4}\ (1 + 4n^2 + 4n)$

$= \frac{n^2\ (n+1)^2}{4}\ (2n + 1)^2$

$= \frac{9.\ n^2\ (n+1)^2\ (2n+1)^2}{36}$

$= 9\left[\frac{n\ (n+1)(2n+1)}{6}\right]^2$

$= 9S_2^2$

= L.H.S.

366 Education Academy

MISCELLANEOUS EXERCISE - 4

1. In a G.P., the fourth term is 48 and the eighth term is 768 . Find the tenth term.

Ans. Given, $t_4 = 48$, $t_8 = 768$

$t_n = ar^{n-1}$

$\therefore t_4 = ar^3$

$\therefore ar^3 = 48$ …(i)

and $ar^7 = 768$ …(ii)

Equation (ii) ÷ equation (i), we get

$\frac{ar^7}{ar^3} = \frac{768}{48}$

$\therefore r^4 = 16$

$\therefore r = 2$

Substituting r = 2 in (i), we get

a. $(2^3) = 48$

$\therefore a = 6$

$\therefore t_{10} = ar^9$

$\therefore t_{10} = ar^9$

$= 6(2^9)$

$= 3072$

2. For a G.P. $a = \frac{4}{3}$ and $t_7 = \frac{243}{1024}$, find the value of r.

Ans. Given, $a = \frac{4}{3}$ and $t_7 = \frac{243}{1024}$

$tn = ar^{n-1}$

$\therefore t_7 = ar^6$

$\therefore \frac{243}{1024} = ar^6$

$\therefore \frac{243}{1024} = \frac{4}{3} r^6$

$\therefore r^6 = \frac{3^6}{4^6}$

$\therefore r = \frac{3}{4}$

3. For a sequence , if $t_n \frac{5^{n-2}}{7^{n-3}}$, verify whether he sequence is a G.P. If it is a G.P. , find its first term and the common ratio.

Ans. The sequence (t_n) is a G.P. if

$\frac{t_{n+1}}{t_n}$ = constant for all n ϵ N.

Now, $t_n = \frac{5^{n-2}}{7^{n-3}}$,

$\therefore t_{n+1} = \frac{5^{n+1-2}}{7^{n+1-3}} = \frac{5^{n-1}}{7^{n-2}}$

$\therefore \frac{t_{n+1}}{t_n} = \frac{5^{n-1}}{7^{n-2}} = \frac{7^{n-3}}{5^{n-2}}$

$= 5^{(n-1)-(n-2)} \times 7^{(n-3)-(n-2)}$

$= 5^{(1)} \times 7^{-1} = \frac{5}{7}$

= constant, for all n ϵ N.

The sequence is a G.P. with common ratio (r) = $\frac{5}{7}$

and first term = $t_1 = \frac{5^{1-2}}{7^{1-3}}$

$= \frac{5^{-1}}{7^{-2}} = \frac{7^2}{5} = \frac{49}{5}$.

4. Find three numbers in G.P. such that their sum is 35 and their product is 1000.

Ans. Let the tree number in G. P. be $\frac{a}{r}$, a, ar.

According to the first condition.

$\frac{a}{r} + a + ar = 35$

$a\left(\frac{1}{r} + 1 + r\right) = 35$ … (i)

According to the second condition,

$\left(\frac{a}{r}\right)(a)(ar) = 1000$

$a^3 = 1000$

$a = 10$

Substituting the value of a in (i), we get

$10\left(\frac{1}{r} + 1 + r\right) = 35$

$\therefore \frac{1}{r} + r + 1 = \frac{35}{10}$

$\therefore \frac{1}{r} + r = \frac{35}{10} - 1$

$\therefore \frac{1}{r} + r = \frac{25}{10}$

$\therefore \frac{1}{r} + r = \frac{5}{2}$

$\therefore 2r^2 - 5r + 2 = 0$

$\therefore (2r - 1)(r - 2) = 0$

$\therefore r = \frac{1}{2}$ or $r = 2$

When $r = \frac{1}{2}$, $a = 10$

$\frac{a}{r} = \frac{10}{\left(\frac{1}{2}\right)} = 20$, $a = 10$ and $ar = 10\left(\frac{1}{2}\right) = 5$

When $r = 2$, $a = 10$

$ar = \frac{10}{2} = 5$, $a = 10$ and $ar = 10(2) = 20$

$\therefore$ the three numbers in G.P. are 20, 10, 5 or 5, 10, 20.

5. Find 4 numbers in G.P. such that the sum of middle 2 numbers is 10/3 and their product is 1.

Ans. Let the four numbers in G.P. be $\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3.

According to the second condition,

$\frac{a}{r^3}\left(\frac{a}{r}\right)(ar)(ar^3) = 1$

$\therefore a^4 = 1$

$\therefore a = 1$

According to the first condition,

$\frac{a}{r} + ar\ \frac{10}{3}$

$\therefore \frac{1}{r} + (1)\ r = \frac{10}{3}$

$\therefore \frac{1+r^2}{r} = \frac{10}{3}$

$\therefore 3 + 3r^2 = 10r$

$\therefore 3r^2 - 10r + 3 = 0$

$\therefore (r - 3)(3r - 1) = 0$

$\therefore r = 3$ or $r = \frac{1}{3}$

When $r = 3$, $a = 1$

$\frac{a}{r^3} = \frac{1}{(3)^3} = \frac{1}{27}$, $\frac{a}{r} = \frac{1}{3}$, ar = 1(3) = 3 and $ar^3 = 1\ (3)^3 = 27$

When $r = \frac{1}{3}$, a = 1

$\frac{a}{r^3} = \frac{1}{(\frac{1}{3})^3} = 27$, $\frac{a}{r} = \frac{1}{(\frac{1}{3})^3} = 3$

$ar = 1\left(\frac{1}{3}\right) = \frac{1}{3}$ and $ar^3 = 1\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

the four numbers in G.P. are

$\frac{1}{27}, \frac{1}{3}$, 3, 27 or 27, 3, $\frac{1}{3}, \frac{1}{27}$.

6. Find five numbers in G.P. such that their product is 243 and sum of second and fourth number is 10.

Ans. Let the five numbers in G.P. be

$\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2

According to the first condition,

$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 243$

$\therefore a^5 = 243$

$\therefore a = 3$

According to the second condition,

$\frac{a}{r} + ar = 10$

$\therefore \frac{1}{r} + r = \frac{10}{a}$

$\therefore \frac{1+r^2}{r} = \frac{10}{3}$

$\therefore 3r^2 - 10r + 3 = 0$

$\therefore 3r^2 - 9r - r + 3 = 0$

$\therefore (3r - 1)(r - 3) = 0$

$\therefore r = \frac{1}{3}, 3$

When $a = 3, r = \frac{1}{3}$

$\frac{a}{r^2} = 27, \frac{a}{r} = 9, a = 3, ar = 1, ar^2 = \frac{1}{3}$

When a = 3, r = 3

$\frac{a}{r^2} = \frac{1}{3}, \frac{a}{r} = 1, a = 3, ar = 9, ar^2 = \frac{1}{3}$

∴ the five numbers in G.P. are

$27, 9, 3, 1, \frac{1}{3}$ or $\frac{1}{3}, 1, 3, 9, 27$

7. For a sequence $S_n = 4\,(7^{n}-1)$ verify whether the sequence is a G.P.

Ans. $\therefore S_{n-1} = 4(7^{n-1}-1)$

But $t_n = S_n - S_{n-1}$

$= 4\,(7^n - 1\,) - 4(7^{n-1}-1)$

$= 4\,(7^n - 1 - 7^{n-1}+1)$

$= 4\,(7^{n-1+1} - 7^{n-1}\,)$

$= 4.7^{n-1}\,(7 - 1)$

$\therefore t_n = 24.7^{n-1}$

$\therefore t_{n+1} = 24\,(7)^{n+1-1}$

$= 24(7)^n$

The sequence (t_n) is a G.P., if $\frac{t_{n+1}}{t_n}$ = constant for all $n \in N$.

$\frac{t_{n+1}}{t_n} = \frac{24\,(7)^n}{24\,(7)^{n-1}}$

$= 7$ = constant, for all $n \in N$.

∴ the sequence is a G.P.

8. Find $2 + 22 + 222 + 2222 + \ldots\ldots$ upto n terms.

Ans. $S_n = 2 + 22 + 222 + \ldots\ldots$ upto n terms

$= 2\,(1 + 11 + 111 + \ldots\ldots$ upto n terms)

$= \frac{2}{9}\,(9 + 99 + 999 +$ upto n terms)

$= \frac{2}{9}\,[(10 - 1) + (100 - 1) + (1000 - 1) + \ldots\ldots$ upto n terms]

$= \frac{2}{9}\,[(10 - + 100 + 100 + \ldots\ldots$ upto n terms) $- (1 + 1 + 1 \ldots..$ n terms)]

Since, 10, 100, 1000, …. n terms are in G.P. with a = 10, $r = \frac{100}{100} = 10$

$\therefore S_n = \frac{2}{9}\left[10\left(\frac{10^n-1}{10-1}\right) - n\right]$

$= \frac{2}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

$\therefore S_n = \frac{2}{81}\,[10\,(10^n - 1) - 9n]$

9. Find the nth term of the sequence 0.6,0.66,0.666,0.6666,…….

Ans. $\therefore t_1 = 0.6$

$t_2 = 0.66 = 0.6 + 0.06$

$t_3 = 0.666 = 0.6 + 0.06 + 0.006$

Hence, in general

$t_n = 0.6 + 0.06 + 0.006 +$... upto n terms.

The terms are in G. P. with

$a = 0.6, r = \frac{0.06}{0.6} = 0.1$

$\therefore t_n =$ the sum of first n terms of the G.P.

$\therefore t_n = 0.6 \left[\frac{1-(0.1)^n}{1-0.1}\right]$

$= \frac{0.6}{0.9}[1 - (0.1)^n]$

$\therefore t_n = \frac{6}{9}[1 - (0.1)^n]$

$= \frac{2}{3}[1 - (0.1)^n]$

10. Find $\sum_{r=1}^{n}(5r^2 + 4r - 3)$

Ans. $= 5\sum_{r=1}^{n} r^2 + 4\sum_{r=1}^{n} r - 3\sum_{r=1}^{n} 1$

$= 5.\frac{n(n+1)(2n+1)}{6} + 4.\frac{n(n+1)}{2} - 3n$

$= \frac{n}{6}[5(2n^2 + 3n + 1) + 12(n + 1) - 18]$

$= \frac{n}{6}(10n^2 + 15n + 5 + 12n + 12 - 18)$

$= \frac{n}{6} = (10n^2 + 27n - 1)$

11. Find $\sum_{r=1}^{n} r(r-3)(r-2)$

Ans. $= \sum_{r=1}^{n}(r^3 - 5r^2 + 6r)$

$= \sum_{r=1}^{n} r^3 - 5\sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r$

$= \frac{n^2(n+1)^2}{4} - 5\frac{n(n+1)(2n+1)}{6} + 6\frac{n(n+1)}{2}$

$= \frac{n(n+1)}{12}[3n(n + 1) - 10(2n + 1) + 36]$

$= \frac{n(n+1)}{12}(3n^2 + 3n - 20n - 10 + 36)$

$= \frac{n(n+1)}{12}(3n^2 - 17n + 26)$

12. Find $\sum_{r=1}^{n} \frac{1^2+2^2+3^2+\cdots+r^2}{2r+1}$

Ans. we know that

$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$\therefore 1^2 + 2^2 + 3^2 + \cdots + r^2 = \frac{r(r+1)(2r+1)}{6}$

$\therefore \frac{1^2+2^2+3^2+\cdots r^2}{2r+1} = \frac{r(r-1)}{6}$

$\therefore \sum_{r=1}^{n} \frac{1^2+2^2+3^2+\cdots+r^2}{2r+1}$

$= \sum_{r=1}^{n} \frac{r(r+1)}{6} = \frac{1}{6}\sum_{r=1}^{n}(r^2 + r)$

$= \frac{1}{6}\left(\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right)$

$= \frac{1}{6}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$

$= \frac{1}{6} \times \frac{n(n+1)}{2}\left(\frac{2n+1}{3} + 1\right)$

$= \frac{n(n+1)}{12}\left(\frac{2n+1+3}{3}\right)$

$= \frac{n(n+1)(2n+4)}{36}$

$= \frac{2n+(n+1)(n+2)}{36}$

$= \frac{n(n+1)(n+2)}{18}$

13. Find $\sum_{r=1}^{n} \frac{1^3+2^3+3^3+\cdots+r^3}{(r+1)^2}$

Ans. $\sum_{r=1}^{n} \frac{1^3+2^3+3^3+\cdots+r^3}{(r+1)^2}$

$\sum_{r=1}^{n} \frac{r^2(r+1)^2}{4} \times \frac{1}{(r+1)^2}$

$= \frac{1}{4}\sum_{r=1}^{n} r^2$

$= \frac{1}{4} \cdot \frac{n(n+1)(2n+1)}{6}$

$= \frac{n(n+1)(2n+1)}{24}$

14. Find $2 \times 6 + 4 \times 9 + 6 \times 12 + \ldots\ldots\ldots$ upto n terms.

Ans. 2, 4, 6,…. Are in A.P.

$\therefore r^{th}$ term $= 2 + (r - 1)\, 2 = 2r$

6, 9, 12, … are in A.P.

$\therefore r^{th}$ term $= 6 + (r - 1)\,(3) = (3r + 3)$

$\therefore\ 2 \times + 6 + 4 \times 9 + 6 \times 12 + \ldots$ upto n terms

$= \sum_{r=1}^{n} 2r \times (3r + 3)$

$= 6\sum_{r=1}^{n} r^2 + 6 \sum_{r=1}^{n} r$

$= 6.\ \frac{n\,(n+1)\,(2n+1)}{6} + 6\,\frac{n\,(n+1)}{2}$

$= n\,(n + 1)\,(2n + 1 + 3)$

$= 2n\,(n + 1)\,(n + 2)$

15. Find $12^2 + 13^2 + 14^2 + 15^2 + \ldots\ldots\ldots\ 20^2$

Ans. $12^2 + 13^2 + 14^2 + 15^2 + \ldots + 20^2$

$(1^2 + 2^2 + 3^2 + 4^2 + \ldots + 20^2) - (1^2 + 2^2 + 3^2 + 4^2 + \ldots\ 11^2)$

$= \sum_{r=1}^{20} r^2 - \sum_{r=1}^{11} r^2$

$= \frac{20\,(20+1)\,(2 \times 20+1)}{6} - \frac{11\,(11+1)\,(2 \times 11+1)}{6}$

$= \frac{20\ \times 21 \times 41}{6} - \frac{11\ \times 12 \times 23}{6}$

$= 2870 - 506 = 2364.$

16. Find $(\ 50^2 - 49^2\ ) + (48^2 - 47^2\ ) + (46^2 - 45^2) + \ldots\ldots\ldots + (2^2 - 1^2\ ).$

Ans. $= (50^2 + 48^2 + 46^2 + \ldots + 2^2) - (49^2 + 47^2 + 45^2 + \ldots + 1^2)$

$= \sum_{r=1}^{25}(2r)^2 - \sum_{r=1}^{25}(2r - 1)^2$

$= \sum_{r=1}^{25} 4r^2 - \sum_{r=1}^{25}(4r^2 - 4r + 1)$

$= \sum_{r=1}^{25}[4r^2 - (4r^2 - 4r + 1)]$

$= \sum_{r=1}^{25}(4r - 1)$

$= 4\ \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$

$= 4 \times \frac{25\,(25+1)}{2} - 25$

$= \frac{4\,(25)\,(26)}{2} - 25$

$= 1300 - 25 = 1275.$

17. In a G.P. if $t_2 = 7$, $t_4 = 1575$ find r

Ans. Given $t_2 = 7$, $t_4 = 1575$

$t_n = ar^{n-1}$

$\therefore\ t_2 = ar$

$\therefore\ 7 = ar$

$a = \frac{7}{r}$ …(i)

$t_4 = ar^3$

$\therefore\ ar^3 = 1575$

$\therefore r^3 \times \left(\frac{7}{r}\right) = 1575$ … [From (i)

$\therefore r^2 \times 7 = 1575$

$\therefore r^2 = \frac{1575}{7}$

$\therefore r^2 = 225$

$\therefore r = \pm 15$

18. Find k so that k-1,k , k+2 are consecutive terms of a G.P.

Ans. Since k – 1, k, k + 2 are consecutive terms of a G.P.

$\therefore \frac{k}{k-1} = \frac{k+2}{k}$

$\therefore k^2 = k^2 + k - 2$

$\therefore k - 2 = 0$

$\therefore k = 2$

19. If p^{th} , q^{th} and r^{th} terms of a G.P. are x, y, z respectively, find the value of $x^{q-r} \times y^{r-p} \times z^{p-q}$

Ans. Let a be the first term and R be the common ratio of the G.P.

$\therefore t_n = a.R^{n-1}$

$\therefore x = a.R^{p-1}$, $y = a.^{q-1}$, $z = a.R^{r-1}$

$\therefore x^{q-r}.\ y^{r-p}.\ z^{p-q}$

$= (a.R^{p-1})^{q-r}.\ (a.R^{q-1})^{r-p}.\ (a.R^{r-1})^{p-q}$

$= a^{q-r}\ R^{(p-1)(q-r).}\ a^{r-p}\ R^{(q-1)(r-p)}.\ a^{p-q}\ R^{(r-1)\ (p-q)}$

$= a^{(q-r+r-p+p-q)}\ .\ R^{[(p-1)]\ (q-r)\ +\ (q-1)\ (r-p)\ +\ (r-1)\ (p-q)]}$

$= a^0\ .\ R^{(pq-pr-q+r+qr+\ -pq-r+p+pr-qr-p+q)}$

$= (1).\ R^0$

$= 1$